THERMOCAPILLARY STABILITY DURING GRAVITY FLOW OF A LIQUID FILM

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Thermocapillary rupture of a film under conditions of turbulent undulatory flow is associated with the buildup of wave motion on its surface. Here an approximate solution to the problem and criterial relations are obtained for determining the limits of stable film flow.

In order to ensure a stable flow without ruptures of a liquid film underheated below its boiling point along a warm vertical surface, it is necessary that the irrigation intensity be higher than a certain minimum Γ_{min} which depends on the thermal flux density [1-3]. Published experimental data [1-3 et al.] cover only a limited range of heat loads, inlet temperatures, and pipe diameters within the active zone. No theoretical generalization has yet been made of the rupture effect in a film underheated below its boiling point under sufficiently high irrigation intensities and thermal flux densities.

This study was conducted with vertical active pipe segments made of stainless steel, ≈1 m long and 10, 15, or 20 mm in diameter. Thermal flux was produced by passing electric current through the channel walls. The experiments were performed under atmospheric pressure. The working liquid (distilled water) was fed through the 1-mm-wide orifice of a film forming device to the outside surface of an active pipe segment and let flow down under the force of gravity. The parameters were varied over the following ranges: inlet temperature of the liquid from 15 to 65°C, thermal flux density from $1\cdot10^4$ to $3\cdot10^5$ W/m^2, and irrigation intensity from 0.1 to 0.7 kg/m·sec.

In each experiment there were established constant thermal flux density and inlet temperature, also a sufficiently high irrigation intensity known to exceed Γ_{min}. The irrigation intensity was then gradually reduced till the appearance of steady dry spots on the active surface. Two irrigation intensities were recorded in the process: Γ_{min} and Γ_1 at which the first dry spot had appeared. The distance from this dry spot to the inlet to the active zone was also recorded.

The graphs in Fig. 1 depict typical results of these experiments made with active pipe segments of various diameters, the data having been presented here in the form of the dependence of Γ_1 on the thermal flux density at various water temperatures ϑ_D, within the rupture region (Fig. 1A), and the dependence of Γ_1 on the water temperature ϑ_D (Fig. 1B). Temperature ϑ_D was determined from the heat balance.

These data indicate, first of all, that Γ_1 within the given range does not depend on the pipe diameter, i.e., on the surface curvature within the active zone.

In the $\Gamma_1 = f(q)$ relation and in the $\Gamma_{min} = f(q)$ relation (the latter not shown) one can distinguish three characteristic ranges. In the first range the necessary minimum irrigation intensity increases with increasing thermal flux density. It covers approximately the range $q \leq 10^5$ W/m^2. There follows the second range, where the necessary minimum irrigation intensity remains almost constant while the thermal flux density increases. With further increase of the thermal flux density, in the third range, the minimum irrigation intensity tends to decrease.

The width of the second range of thermal flux densities, where $\Gamma_1 = \text{const}$, depends on the water temperature and decreases as the latter rises. At a water temperature of 75-85°C, e.g., the irrigation intensity Γ_1 decreases distinctly as the thermal flux intensity q increases above $1.2\cdot10^5$ W/m^2.

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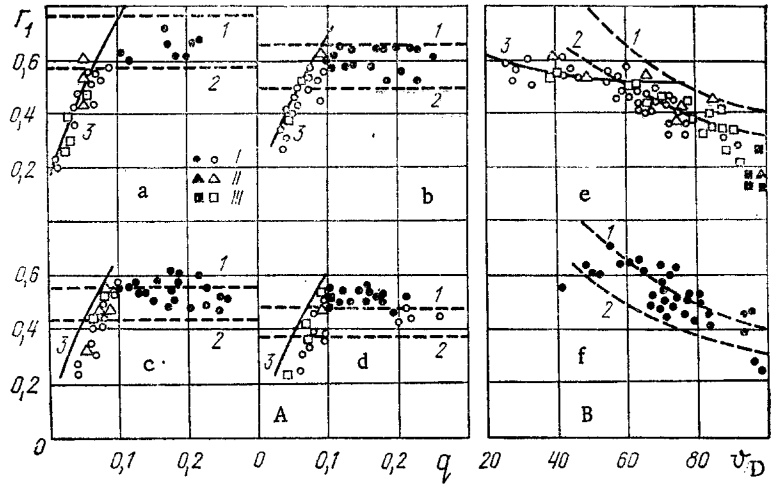

Fig. 1. Dependence of the necessary irrigation intensity Γ_1, kg/m·sec, on the thermal flux density q, MW/m^2 (A) and on the temperature ϑ_D (B), within the rupture region: I) diameter 10 mm, II) diameter 15 mm, III) diameter 20 mm; (a) $\vartheta_D = 45$–55°C and 1) $N_{Re,\delta_c} = 300$, 2) $N_{Re,\delta_c} = 200$, 3) $\vartheta_{calc} = 50$°C; (b) $\vartheta_D = 55$–65°C and 1) $N_{Re,\delta_c} = 300$, 2) $N_{Re,\delta_c} = 200$, 3) $\vartheta_{calc} = 60$°C; (c) $\vartheta_D = 65$–75°C and 1) $N_{Re,\delta_c} = 300$, 2) $N_{Re,\delta_c} = 200$, $\vartheta_{calc} = 70$°C, (d) $\vartheta_D = 75$–85°C and 1) $N_{Re,\delta_c} = 300$, 2) $N_{Re,\delta_c} = 200$, $\vartheta_{calc} = 80$°C; (e) $q = 0.04$–0.08 MW/m^2 and 1) $N_{Re,\delta_c} = 300$, 2) $N_{Re,\delta_c} = 200$, 3) $q_{calc} = 0.06$ MW/m^2; (f) $q = 0.12$–0.16 MW/m^2 and 1) $N_{Re,\delta_c} = 300$, 2) $N_{Re,\delta_c} = 200$.

The absolute value of the ultimate Γ_1 and Γ_{min} levels within the second range also depends on the water temperature and decreases as the latter rises. In all cases the necessary minimum irrigation intensity decreases as the temperature rises within the rupture region.

On the basis of the temperature within the rupture region (Fig. 1B), one can tentatively distinguish two ranges of the $\Gamma_1 = f(q)$ relation. In the first range (at relatively low temperatures) Γ_1 decreases only insignificantly as the temperature rises. In the second range Γ_1 decreases fast as the temperature rises. At temperature $\vartheta_D > 90$°C the decrease of Γ_1 in some cases occurs again at a slower rate.

Continuity of film flow under thermal flux densities lower than $\approx 10^5$ W/m^2 and at true temperatures lower than ≈ 90°C within the rupture region was broken in all experiments at a distance of 0.4–0.8 m from the inlet, the location of the film rupture depending neither on the thermal flux density nor on the water temperature at the inlet and the temperature at the site of film rupture. Although the coordinates of the rupture point spread rather widely, this spread is a random one. This agrees with the data in another study [2]. At high thermal flux densities and true temperatures there occurs a transition to bubble boiling of the film. This begins at the outlet from the active zone and ascends upstream as the thermal flux density is increased. In this case a dry spot forms at the boundary between boiling and nonboiling film, it also ascends as the thermal flux density is increased. The black dots around experimental points in Fig. 1 correspond to this condition.

Breakdown of a liquid film under nonisothermal conditions is usually attributed to thermocapillary effects at its surface. The gist of this phenomenon is as follows.

As a liquid film of nonuniform thickness is heated, a temperature gradient builds up on its surface, whichproduces a gradient of surface tension. The latter gives rise to tangential surface stresses which cause the liquid in the film to flow from regions where the surface tension is lower (where the temperature is higher in the case of liquids with a negative temperature gradient of surface tension) to regions where the surface tension is higher (lower temperature). Such effects can be significant only in the film layer where the flow

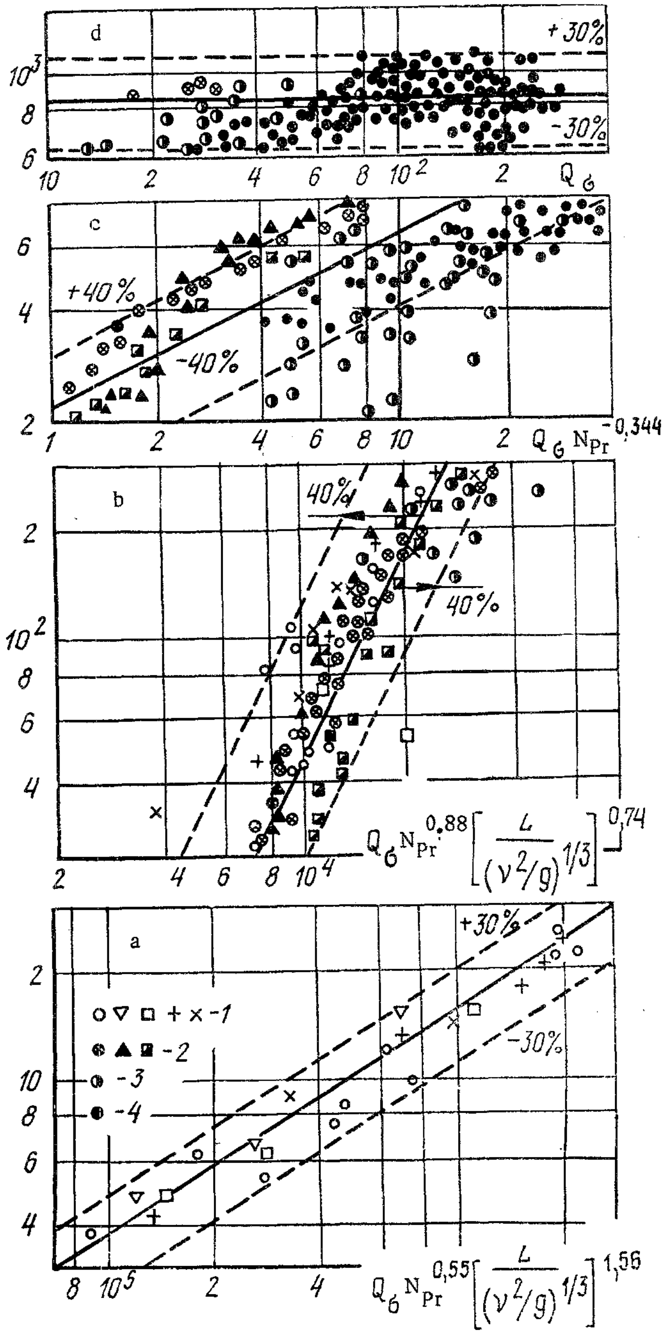

Fig. 2. Criterial generalization of experimental data: 1) [1], 2) [2], 3) [3], 4) this study.

is laminar. Accordingly, mathematical models involving the mean film thickness (such as the model in [1]) yielda sufficiently close agreement with experimental data only within the range of low irrigation intensities ($\Gamma_{min} < 0.1$ kg·m⁻¹·sec⁻¹).

Our study (just as the data in [2, 3]) reveals that instability due to increasing thermal flux intensity can also occur at much higher irrigation intensities, i.e., in the range of turbulent undulatory flow. It is well known that on the film surface there develops a wave motion with a fast increasing amplitude at the beginning of the active zone [4, 5]. Between waves there build up thin layers of liquid, a so-called continuous layer. Large waves follow one another at a definite frequency and they carry a large part of the flow mass. The surface between them is covered with small capillary waves. The thickness of the continuous layer can be sufficiently small so that the molecular viscous forces will predominate in it, ensuring a laminar flow and making the occurrence of thermocapillary effects possible.

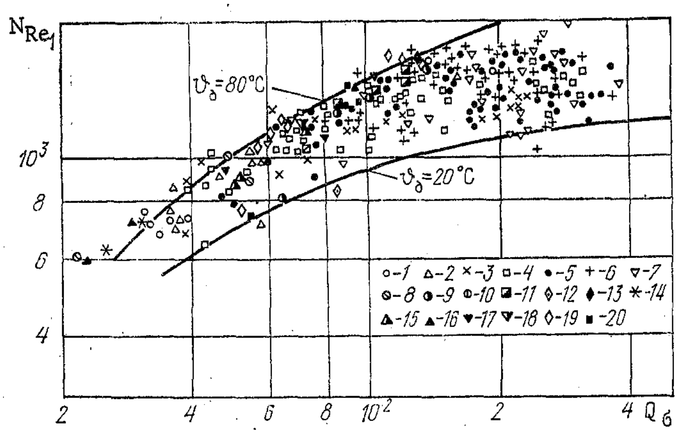

Fig. 3. Relation $N_{Re,1} = f(0_\sigma)$ in the third range: diameter 10 mm and ϑ_D, °C: 1) 25-35; 2) 35-45; 3) 48-55; 4) 55-65; 5) 65-75; 6) 75-85; 7) 85-100; diameter 15 mm and ϑ_D, °C: 8) 35-45; 9) 45-55; 10) 55-65; 11) 65-75; 12) 75-85; 13) 85-100; diameter 20 mm and ϑ_D, °C: 14) 25-35; 15) 35-45; 16) 45-55; 17) 55-65; 18) 65-75; 19) 75-85; 20) 85-100.

Estimates of the thickness of the continuous layer in all cases of film rupture indicate that the flow in films is rather laminar undulatory. It has been found in earlier studies [6] that the flow in a film remains fully laminar up to a Reynolds number of ≈200. The dashed lines on the diagram in Fig. 1 indicate irrigation intensities corresponding to $N_{Re,\delta_c} = 200$ and 300, respectively. Here the Reynolds number N_{Re,δ_c} has been calculated from the limiting thicknesses of the continuous boundary layer. According to the graphs, in the $q < 10^5$ W/m² range film rupture occurs at $N_{Re,\delta_c} < 200$-300.

The wave motion is an ergodic weakly stationary random process. Consequently, the point corresponding to the true minimum thickness of the continuous layer does not have a fixed location and can shift within a certain segment along the film path, but remains sufficiently far away from the inlet. This indeed relates to the experimentally established fact that a dry spot had formed within the 0.4-0.8 m range of distances from the inlet, independently of the thermal flux density and of the water temperature.

Accordingly, the buildup of thermocapillary effects during turbulent undulatory downward flow of a film has to do with the buildup of wave motion at its surface. Rupture of the film will occur if thin layers of laminarly flowing liquid appear between large waves and if the time necessary for thermocapillary rupture is shorter than the time between two successive large waves passing over a monitored film segment with a thin continuous layer.

The qualitative analysis of film rupture under a thermal flux makes it possible now to proceed with a simplified mathematical description of the said phenomenon.

We will make the following assumptions regarding the mechanism of film rupture:

1. The cause of local film thinning is the thermocapillary effect.
2. The process of film rupture occurs in two stages: 1) local thinning and 2) sudden film rupture after the critical fimlm thickness δ_{cr} corresponding to rupture under isothermal conditions has been reached at the location of most appreciable thinning down.
3. Thermocapillary flow is caused by tangential stress [1 et al.], i.e., $\tau = \mathrm{grad}\, \sigma$.
4. The region of likely film rupture is a trough between two large waves.
5. The temperature of the liquid in a wave and the initial temperature in a trough are equal to the mean temperature calculated from the equation of heat balance.
6. Sinusoidal capillary waves exist at the film surface over a trough.
7. Under consideration is only the secondary flow, called the thermocapillary effect, viz., the relative motion in a system of coordinates which moves at the velocity of gravity flow of the liquid over a wave through.

We consider the plane problem of a liquid heated and moving in a capillary wave, in a system of coordinates x, y which moves together with the trough of a large wave. The equations of motion and energy in the boundary-layer approximation are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial y^2}, \tag{1}$$

$$\frac{\partial \vartheta}{\partial t} + u\frac{\partial \vartheta}{\partial x} = a \frac{\partial^2 \vartheta}{\partial y^2}. \tag{2}$$

The equation of the film surface is

$$\frac{\partial \delta}{\partial t} = -\frac{\partial}{\partial x}\int_0^\delta u\,dy. \tag{3}$$

Initial and boundary conditions for Eqs. (1)-(3) are

$$\text{at} \quad t=0 \quad \vartheta=\bar{\vartheta},\ u=0;\ \delta=\delta_0\left(1-\alpha_0\cos\frac{2\pi x}{l_0}\right); \tag{4}$$

$$\text{at} \quad y=0 \quad -\lambda\frac{\partial \vartheta}{\partial y}=q;\ u=0; \tag{5}$$

$$\text{at} \quad y=\delta \quad \frac{\partial \vartheta}{\partial y}=0;\ \mu\frac{\partial u}{\partial y}=\tau_x=\frac{\partial \sigma}{\partial x}=\frac{d\sigma}{d\vartheta}\frac{\partial \vartheta}{\partial x}; \tag{6}$$

$$\text{at} \quad x=0 \quad \frac{\partial \delta}{\partial x}=0;\ \text{at} \quad x=\frac{l_0}{2} \quad \frac{\partial \delta}{\partial x}=0. \tag{7}$$

For solving the system of equations (1)-(3) we will make several additional assumptions.

Convective heat transfer will be disregarded in Eq. (2). This will result in a one-dimensional equation of transient heat conduction through a liquid film of thickness δ. The quasisteady solution to this equation is [7]

$$\vartheta = \bar{\vartheta} + \frac{q\delta}{\lambda}\left\{\frac{at}{\delta^2} + \frac{1}{2}\left[\left(\frac{\delta-y}{\delta}\right)^2 - \frac{1}{3}\right]\right\}. \tag{8}$$

Relation (8) is valid at time $t \geq t_p$. The error of solution (8) decreases with increasing time and asymptotically approaches zero.

We will confine the solution to the region $(0,\ 0+\Delta x)$, where $\Delta x \ll l_0$. Let the wall temperature within the region Δx be a function of time only. The temperature of the liquid at the film surface can then be expressed as

$$\vartheta(\delta,\ t) = \vartheta_w(t) - \Delta\vartheta(\delta), \tag{9}$$

where the difference between wall temperature and surface temperature can be determined with the aid of relation (8) as

$$\Delta\vartheta(\delta) = \vartheta(0,\ t) - \vartheta(\delta,\ t) = \frac{q\delta}{2\lambda}. \tag{10}$$

Inserting expression (10) into Eq. (9) and then differentiating the latter with respect to x yields

$$\frac{\partial \vartheta}{\partial x} = -\frac{q}{2\lambda}\frac{\partial \delta}{\partial x}. \tag{11}$$

We assume that the velocity distribution of thermocapillary flow is at every instant of time determined by the quasisteady Couette flow under a given tangential surface stress, viz.,

$$u(x,\ y,\ t) = \frac{\tau_x(x,\ t)}{\mu}y. \tag{12}$$

Inserting expression (12) into Eq. (3) and substituting for τ_x its value from boundary condition (6) yields, with the aid of relation (11),

$$\frac{\partial\delta}{\partial t}+\frac{q}{4\lambda\mu}\left(-\frac{d\sigma}{d\vartheta}\right)\left[\delta^2\,\frac{\partial^2\delta}{\partial x^2}+2\delta\left(\frac{\partial\delta}{\partial x}\right)^2\right]=0. \tag{13}$$

This equation describes the deformation, with time, of the profile of a capillary wave. Assuming that the surface curvature within the given region $(0,\ 0+\Delta x)$ remains constant throughout the entire thinning process and equal to what it was initially, which is valid in the case of an invariable dimensionless wavelength, we have

$$\frac{\partial^2\delta}{\partial x^2}=\left.\frac{\partial^2\delta}{\partial x^2}\right|_{t=0}=-\alpha_0\delta_0\left(\frac{2\pi}{l_0}\right)^2\cos\frac{2\pi x}{l_0}. \tag{14}$$

Since $\Delta x \ll l_0$ by definition, hence $\cos(2\pi/l_0)\simeq 1$ and

$$\frac{\partial^2\delta}{\partial x^2}=-\alpha_0\delta_0\left(\frac{2\pi}{l_0}\right)^2. \tag{15}$$

When $\Delta x \ll l_0$, on the other hand, then

$$\frac{\partial\delta}{\partial x}=\alpha_0\delta_0\,\frac{2\pi}{l_0}\sin\frac{2\pi x}{l_0}\simeq 0. \tag{16}$$

Introducing the dimensionless wavelength $\bar{l}_0 = l_0/\delta_0$ into expression (15) and then inserting the latter into Eq. (13) yields, with the aid of relation (16), the solution for the film thickness within the trough of a capillary wave

$$\frac{1}{\delta(x,\ t)}=\frac{1}{\delta(x,\ t_p)}+\pi^2\,\frac{\alpha_0(1-\alpha_0)}{\bar{l}_0^2}\,\frac{q(-d\sigma/d\vartheta)}{\lambda\mu\delta_c}\,(t-t_p). \tag{17}$$

From expression (17) we in turn obtain one for the time t_{cr} during which film thickness within a wave trough at x = 0 decreases from the initial δ_c to δ_{cr}, at which rupture occurs, viz.,

$$\Delta t_{cr}=\frac{1}{\pi^2}\,\frac{\bar{l}^2}{\alpha_0(1-\alpha_0)}\,\frac{\lambda\mu\delta_c}{q(-d\sigma/d\vartheta)}\left(\frac{1}{\delta_{cr}}-\frac{1}{\delta_c}\right). \tag{18}$$

The time of thermocapillary buildup within a wave trough consists of the time t_p till the temperature of the film surface begins to change and the time Δt_{cr} during which the film thins down. The condition for film rupture within a wave trough is

$$t_p+\Delta t_{cr}\leqslant\frac{1}{\omega}. \tag{19}$$

The second condition for film rupture can be written as

$$Re_{\delta_c}\leqslant Re_{lim}. \tag{20}$$

On the basis of earlier results [4], we stipulate* that $N_{Re,lim} = g\delta^3/3\nu^2 = 200$–$300$.

On the basis of solution (8), we evaluate t_p as

$$t_p=\frac{\delta_c^2}{6a}. \tag{21}$$

The magnitude of δ_{cr} can be found from the expression $N_{Re,cr} = g\delta_{cr}^3/3\nu^2 = 4.18$ [8].

A comparison of experimental data and calculations has shown that the value of the dimensionless group $\bar{l}_0^2/\alpha_0(1-\alpha_0)$ can be treated as a constant equal to $2.5\cdot10^4$. (According to the data in study [9], the value of this quantity is approximately $2\cdot10^4$). The final expression for Δt_{cr} will be

$$\Delta t_{cr}=\frac{2.5\cdot10^4}{\pi^2}\,\frac{\lambda\mu}{q(d\sigma/d\vartheta)}\,\frac{\delta_c-\delta_{cr}}{\delta_{cr}}.$$

The curves in Fig. 1 have been calculated according to these relations, assuming the conditions (19) and (20) to be satisfied. Some discrepancy between experimental data and calculated values in the range of high temperatures and thermal flux densities is apparently due to an additional effect of bubble boiling on the stability of a film.

We will now perform a criterial analysis of our experimental data and compare them with already published ones.

*Reynolds number is denoted by N_{Re} in text and Re in display equations throughout etc.

From the system of equations (1), (2), and (6) with appropriate scales for the quantities involved, we obtain dimensionless groups of independent determining quantities:

$$\frac{at_{cr}}{\delta_c^2}=\mathrm{Fo};\quad \frac{\nu}{a}=\mathrm{Pr};\quad \frac{q\,(d\sigma/d\vartheta)\,\delta_c^2}{\lambda\rho\nu^2}=Q_{\sigma 1}. \tag{23}$$

We use the following scales: $\Delta\vartheta=\vartheta_w-\bar{\vartheta}_c$ for the temperature, the mean (over the thickness of the continuous layer) velocity of thermocapillary flow $\bar{u}$ for the velocity, t_{cr} for time, and δ_c for the linear dimensions.

The solution to the system of equations (1)-(3) will be sought in the form

$$Q_{\sigma 1}=C\,\mathrm{Pr}^m\mathrm{Fo}^k. \tag{24}$$

Considering that [3] in the range of turbulent undulatory flow of a film

$$t_{cr}=\frac{1}{\omega}\sim\left(\frac{\nu}{g^2}\right)^{1/3}\mathrm{Re}_{min}^{n_2}; \tag{25}$$

$$\delta_c\sim\left(\frac{\nu^2}{g}\right)^{1/3}\mathrm{Re}_{min}^{n_3}, \tag{26}$$

we obtain from relation (24) with the criterial numbers N_{Fo} and $Q_{\sigma i}$ transformed

$$\mathrm{Re}_{min}=C_1Q_\sigma^n\mathrm{Pr}^m, \tag{27}$$

where $Q_\sigma=q(d\sigma/d\vartheta)/\lambda\rho g(\nu^2/g)^{1/3}$ is the criterial number for the thermocapillary stability.

In the range of laminar undulatory flow without large waves

$$t_{cr}=\frac{L-L_0}{\bar{w}}, \tag{28}$$

$$\bar{w}\sim(\nu g)^{1/3}\mathrm{Re}_{min}^{n_4}. \tag{29}$$

$$\delta_c=\bar{\delta}\sim\left(\frac{\nu^2}{g}\right)^{1/3}\mathrm{Re}_{min}^{n_5}. \tag{30}$$

Relations (28)-(30) yield

$$\mathrm{Fo}=\mathrm{Pr}\frac{L-L_0}{(\nu^2/g)^{1/3}}\,\mathrm{Pr}_{min}^{n_5}. \tag{31}$$

Accordingly, for the laminar turbulent range Eq. (24) becomes

$$\mathrm{Re}_{min}=C_2Q_\sigma^{n_1}\left[\frac{L-L_0}{(\nu^2/g)^{1/3}}\right]^{m_1}\mathrm{Pr}^{k_1}. \tag{32}$$

The criterial number $Q_\sigma=q(d\sigma/d\vartheta)/\lambda\rho g(\nu^2/g)^{1/3}$ is widely used [1-3] in analytical solutions and for evaluation of experimental data.

The data of this study as well as those from the three other studies [1-3] are shown in Fig. 2 with Q_σ as the independent variable.

The results of this study correlate closely with the results in [3] over the given range of comparison and then substantially extend the range of thermal flux densities.

In the $N_{Re,min}=f(Q_\sigma)$ relation one can distinguish four characteristic ranges.

The first range is that of small $N_{Re,min}$ ($N_{Re,min}<25$) and Q_σ ($Q_\sigma<1$). Here $N_{Re,min}$ increases rather slowly with increasing Q_σ, the relation depending appreciably on the length of the active zone and on the properties of the liquid. The experimental data here fit, within a ±25% accuracy, the equation (Fig. 2a)

$$\mathrm{Re}_{min}=2.3\cdot 10^{-3}Q_\sigma^{0.65}\mathrm{Pr}^{0.35}\frac{L}{(\nu^2/g)^{1/3}}. \tag{33}$$

In the second range ($25<N_{Re,min}<200$ and $1<Q_\sigma<2.5$) the dependence of $N_{Re,min}$ on Q_σ is much stronger, the relation also depending appreciably on the length of the active zone and on the properties of the liquid. The experimental data fit, within a ±40% accuracy, the equation (Fig. 2b)

$$Q_\sigma = 62.5\mathrm{Re}_{\min}^{0.59}\mathrm{Pr}^{-0.88}\left[\frac{L}{(\nu^2/g)^{1/3}}\right]^{-0.74} . \quad (34)$$

In the third range ($300 < N_{\mathrm{Re,min}} < 700$ and $25 < Q_\sigma < 50$) the dependence of $N_{\mathrm{Re,min}}$ on Q_σ is weaker, with the length of the active zone and the properties of the liquid playing no detectable role. The experimental data fit rather closely (within ±40%) the equation [3] (Fig. 2c)

$$\mathrm{Re}_{\min} = 217Q_\sigma^{0.47}\mathrm{Pr}^{-0.162} . \quad (35)$$

In this third range the same data, but referred to the temperature at the rupture section rather than to the inlet temperature as in Eq. (35), fit very accurately into the generalization developed in this study by the method of analysis described here (Fig. 3).

In the fourth range ($N_{\mathrm{Re,min}} > 700$ and $Q_\sigma > 50$) the value of $N_{\mathrm{Re,min}}$ depends neither on Q_σ nor on the length of the active zone and the properties (temperature) of the liquid. Accurately within ±30% (Fig. 2d), $N_{\mathrm{Re,min}} = 850$ or $N_{\mathrm{Re,min}} < 1150$.

NOTATION

$\Gamma_{\min}$, kg/m·sec, minimum irrigation intensity at which no film rupture occurs; Γ_1, kg/m·sec, irrigation intensity at which the first dry spot appears; q, W/m², thermal flux density; ϑ_D, °C, temperature at the rupture section; x, m, space coordinate along the warm surface in the direction of flow; y, m, coordinate in the direction normal to the warm surface; δ_0, m, mean thickness of the film between large waves; δ_c, m, thickness of the continuous layer; δ_{cr}, m, critical film thickness; $\alpha_0 = \alpha/\delta_0$ and $\bar{l}_0 = l_0/\delta_0$, respectively, dimensionless initial amplitude and length of a wave; ω, sec^{-1}, recurrence frequency of large waves; t_{cr}, sec, time till thermocapillary rupture of a film; t_p, sec, time of penetration of a thermal perturbation through the film thickness; $\Delta t_{cr} = t_{cr} - t_p$; u, m/sec, velocity of thermocapillary flow of the liquid; λ, W/m·°C, thermal conductivity; c_p, kJ/kg·°C, specific heat; ρ, kg/m, linear density; μ, N·sec/m², dynamic viscosity; a, m²/sec, thermal diffusivity; σ, N/m, surface tension; τ, N/m², tangential stress at the film surface; L, m, length of the warm pipe segment; L_0, m, distance from the inlet to the section where wave motion at the film surface occurs; $\bar{w}$, m/sec, mean velocity of downward flow of liquid in the film; $\bar{\delta}$, m, mean thickness of the laminar layer; and g, m²/sec, free-fall acceleration due to gravity.

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